STUDENT AND TEACHER AFFECT

The goals of enjoying mathematics and being persistent are of paramount importance to a student's success and should be primary considerations in the design of class activities. The goals of a research experience are for students to engage in lengthy, complex projects; to take risks and grapple with deep ideas; and to be sufficiently satisfied with the experiences so that they want to face such difficulties again in their lives. Realizing these goals requires tenaciousness and a tolerance for frustration. Students (and adults) can become paralyzed, depressed, or antagonistic when faced with open-ended, incompletely defined tasks. While lessons should gradually increase in the level and length of challenges involved, no amount of easing in can eliminate the symptoms just cited. It is essential to make clear to the students what they will face, how they may feel, why you are asking them to experience certain frustration, and how you will support them throughout the process.

Student Expectations

Many students believe that a teacher's job is to explain everything for them—to turn complicated ideas into simple, predigested ones. Students and teachers alike are uncomfortable with the possibility that a teacher could answer a question with "I don't know." When students are engaging in real research—when they are immersing themselves in the *doing* of a discipline—your role is not to be the Grand Explainer. It is important to tell your students that your job is to help them find interesting avenues for investigation and to help them learn how to overcome the many intellectual obstacles they will encounter in their work. Your job is to provide a framework for approaching new situations, to ask the questions the students need to learn how to ask themselves. A Making Mathematics mentor encouraged their teacher mentee with the following note:

> So much of the stuff you write about—"they no longer needed me to validate their work... they saw how much more complicated the problem became..."—that's all so important in understanding how mathematics really works. If you're trying to work on a problem that's new, no one **can** validate your work because you're the first one to try it! You **have** to figure it out, convince yourself, and then convince others. I also like that you reinforced how much

> progress they had made since starting to work on the project; kids don't often see that kind of progress around a single problem—a noticeable, measurable increase in what they're able to do with it. That's really cool.

Why should students want to undertake such a perilous voyage with guarantees of hardship? Because it is in the overcoming of those impediments that deep understanding emerges. To demonstrate, ask students to add 3 + 5 and then ask them how much they learned from that problem. Learning occurs when we try new challenges that force us to discover new ideas or make new connections between old ideas.

Many students think that a teacher who will not explain everything is leaving them to their own devices. On a regular basis, remind your students (or a recalcitrant individual) that you do not expect them to sink or swim on their own. Encourage (or shanghai) them to come to ask questions about a problem. Some students have difficulty assessing when they are really stuck, some think they are always stuck (and ask for help too quickly), and others think asking for help is wrong. It is sometimes difficult to distinguish for the students and oneself where the line between increasing independence begins and avoiding floundering ends. Err on the side of giving extra help, so long as that help continually models the internal dialogue that problem-solvers need. When a student is stuck, share with them the questions that you would ask yourself in order to solve the problem. Ask your students to think about how your questions are a response to a particular situation and encourage them to ask themselves questions about the problems that they tackle (see Getting Stuck, Getting Unstuck).

Advice For Long-Term Class Research

Students are much more open to meeting unsettling expectations if they, in turn, have some control over how the course is structured. Students are understandably fearful of failure. Negotiating due dates for major assignments can help students feel confident that they will have time to do the assignment well. It is also reasonable to offer extensions when students can demonstrate that they worked hard on a problem but would like to try to make a little more progress. Requiring that extensions be requested at least a day in advance discourages last-minute efforts and adds the minimal requirement that students realize when they are not progressing as quickly as they had hoped—a step students need to take when monitoring the progress of lengthy projects. Evidence of effort should consist of more than wracked brains—they should provide substantial logbook work.

The above policies reduce the stress associated with research deadlines when these deadlines may not conform to the particular time needs of a student and his or her project. Creative breakthroughs are difficult to produce on a schedule, but, with persistence, they should occur, and it is important to reduce the sense of risk associated with these uncertainties. Weekly touch-base sessions that allow students or research groups the chance to provide updates on their progress help students avoid last-minute "cramming". Additionally, these sessions give the teacher a chance to ask clarifying questions about the work undertaken thus far and reduce the stress some students feel when asking for individual help outside of class. Written updates, such as rough drafts or short progress reports from the students, are also useful for scaffolding students' use of time.

Student Interests

Another way teachers give students control of a course is through the attention we pay to their ideas, questions, and interests. This attention is central to meeting the goals that students be investigative, make connections, and identify topics for exploration. Students only embrace these goals if the sought-after behaviors get the response they deserve, namely, class discussions and assignments that move in the direction of their thoughts and interests. For example, in one of our classes, as a result of student questions, the students spent two weeks studying game theory and the Prisoner's Dilemma. This seeming digression spurred many students on to ask for readings on mathematics topics of individual interest to them.

Although the thought of having students choose course content may disturb some, this choosing is a clear indication that students are developing particular mathematics interests and actually want to do math. However, this approach means that heavily detailed syllabi become either useless (as you would need to update the syllabus daily, or ignore students' comments that did not head in the pre-planned direction) or destructive (students who see content-laden syllabi quickly shut down their own initiative and mathematical problem-posing). Just as teachers have their own mathematical aesthetics, so do students. We all have favorite topics and activities that get us really excited. Both variety and flexibility are essential to maintaining this built-in source of motivation.

Change Takes Time

The development of persistence in both individuals and a class requires careful planning. It is important that students have some early successes, but these need not always be immediate or complete. Challenges of greater length and difficulty should be introduced gradually, and students need to have most of the skills and understandings required to solve a problem (with just one or two gaps, which may be the sought-after goal of the activity). Work on projects follows rhythms (of effort, insight, and types of tasks) that are not present in shorter exercises and to which it takes time to adapt. Both the teacher and students need to be patient with these changes.

If you are only spending a few class periods introducing research ideas (e.g., using the Introductory Explorations activity, a warm-up problem from a project, or Trains), you may discover the frustration that students have when they are stymied by a problem and you may not have the time to help them master all of the emotional and mathematical understandings that will make them want to persist with research. Try to pick content that is familiar to students so that the focus can be on the research ideas themselves (e.g., the role of conjectures). Have reasonable expectations for a first research experience. Return to research activities periodically so that students will have practice that develops their understanding of the research process and builds their confidence that they can stick with a problem until they have made progress with it.

Teacher Affect

The most important determinant of the students' affect will be the teacher's. We must be excited about mathematics and share that enthusiasm unashamedly with our students. Students will jump in when they see that you are interested in their questions, work on their problems, and bring them articles and books on topics of interest. You will be most effective if you are willing to get happily perplexed by difficult problems in front of your classes. No one expects figure skaters to divine the proper position for a leap; they are shown one in action. Similarly, teachers as coaches must be mathematicians in front of their students, willing to take an occasional "fall on the ice." Students will be impressed if you can show your excitement about the process of exploring an intriguing conjecture whose truth you do not know. If you get stuck, continue to work on the problem on your own and bring in any results or your latest dead-ends until you have made some progress. You can also consult a colleague or book and cite your research. Be sure to allow students to tackle problems even if you do not know the answer in advance. If kids see that you are willing to let go of a safety net, then they will be too.

The best way to prepare for doing mathematics investigations with students is to do mathematics research oneself. Doing research hones our ability to identify different choices and make decisions throughout the research process. It also makes us more sensitive to the emotional tumult that open-ended research evokes in students and adult mathematicians alike. Consider the following comments from two Making Mathematics teachers working with other teachers on a research question via email. The first quote is a wonderfully honest description of a fairly typical reaction to a new problem:

> After receiving the first communication I have to tell you...I was ready to quit. I read the assignment and my eyes crossed. Panic. Overwhelmed. Then I thought of all my students (8th grade, critical year for self-concept as a math student) who have to live this all the time—don't understand as quickly as their peers, can't get it in the first reading—and I decided to continue. If nothing else (and, of course, there will be a lot more), my empathy meter will be higher next year. So, I read and reread the email, checked the suggested links, started to get a handle on it and the panic subsided. I'm here for the duration.

This teacher went on to pose many interesting problems that expanded the group's research agenda. Emotions while doing research often continue to swing wildly even after that first fearful start. Consider this teacher's roller coaster:

> Early on: I'm getting a real feel of what it is like to wrestle with a chart of numbers and try to make some sense of it. My wife keeps yelling to me from downstairs "It's summer; put the math away!" I don't know how much longer I can keep my enthusiasm up without a breakthrough. For now, I'm going to give it a rest. I have been noticing how I go about dealing with things though. I really like to make use of technology where I can (realizing it doesn't constitute a proof; but it's sure nice to have lots of data to

> work with.) I'm also pretty visual. I started using colored pencils right away.
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> Two days later after a breakthrough: Anyhow, I felt I accomplished something today. I too feel guilty when I'm not inspired to move on or when I get discouraged. I keep thinking, how will I get my students to stay on task if I wander myself?
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> A few days later: I started working with the actual numbers to see if I could find a functional relationship (I tend to head towards functions right away, because I do a lot with them in the courses I teach.) This just overwhelmed me. I made two columns "Row #" and "Number of entries that are divisible by 3." I could see this wasn't going to be easy for me to do. In short, I'm stuck and feeling lost. I'm OK with experiencing these emotions, because I want to be able to empathize with my students. But I must say, I'm feeling like a wimp right now, because I want to say this is too hard.

This teacher made a number of discoveries that he was able to prove. Typical of successful research, his progress often followed periods of intense immersion coupled with lulls that allowed his thoughts to settle.

Doing research does not always make us feel more confident about our mathematical skills. The more challenges we take on, the more we learn about the extraordinary depth and breadth of the subject—we learn how much we do not know and how difficult the field can be. Strive to let that knowledge inspire you with the possibilities that lie ahead for you, your students, and for mathematics itself. Teachers often fear being asked questions in class that they cannot answer, but being experienced in mathematics and being all-knowing are not the same thing.

I find it liberating to be able to answer a question with "I don't know!" I am not pretending not to know, so I am that much more able to model good problem solving. I think the lack of certainty in such circumstances is very exciting—it is real math! Science experiments where the teacher always knows what should happen can seem forced. Real science is about new questions, failed experiments, and messy data. Mathematics should be full of the same dead ends and surprising discoveries. The alternative to admitting our occasional ignorance is saying that mathematics is a small set of "school" skills that we have mastered and that our students must learn. That is not a fun, intellectually interesting, or educationally useful choice.

Final Thoughts

There is an additional benefit and challenge posed by the strong feelings of joy and frustration elicited by open-ended research. They force us to acknowledge more of each student's personality. We have to accept their individual responses to frustration and help them both adjust and learn to cope with these emotions. This necessity highlights how curricular reform often forces multiple changes because of the complexity of good teaching and learning.

One barrier to engaging students in mathematics is the notion, common in the United States, that our discipline is somehow special and that real work in mathematics requires a talent that few possess (see Coping With Math Anxiety for further discussion). You can help dissolve this sense of a "high priesthood" by referring to your students as mathematicians and by helping your class see itself as a mathematical community in search of discoveries. This language will complement peer review activities (see Feedback in the Assessment section, and Writing Conjectures in the Conjectures section) and help the students recognize that there is an audience for their efforts beyond their teacher.

Portions of this document are adapted from Mathematical Modeling: Teaching the Open-ended Application of Mathematics © Joshua Abrams 2001 (available at http://www.meaningfulmath.org/modeling*) and used with permission.*

BIBLIOGRAPHY

Coping With Math Anxiety is available online at http://www.mathacademy.com/pr/minitext/anxiety/index.asp.