RESOURCES

There are two meanings to the word “research”—one is an investigation that establishes a result. The other is “researching a topic” in the sense of gathering information. This handbook is about the former, but the latter, which this section addresses, is important, too.

Teachers and students engaged in research need ready access to mathematics resources. An investigation may take us into areas of mathematics that are not entirely familiar. When it does, we need to read up on the topic and learn enough to determine if the ideas might be of use in our work. For example, a student in a mathematics research seminar investigated peg solitaire games. He found several journal articles about these games that used methods based on group theory. In order to understand and apply the ideas from the published literature, he needed to find out how groups were defined and what their basic properties were. He was then able to adapt the methods of earlier researchers to his particular problems.

As the above example suggests, it is neither possible nor advisable for students to have to rediscover, and make sense of, all mathematics on their own. They do need to recognize when learning new content might be helpful and how to obtain and use the resources that are available (see Getting Information). As students apply the results that they encounter in their readings or that their peers produce, they come to recognize the cumulative and communal nature of mathematics. Interesting mathematics is almost always the product of building on the work of others.

When students spend time browsing through mathematics books, magazines, and web sites, they gain a better appreciation for the breadth, depth, and open-ended nature of the discipline than textbooks provide. Below are listed materials that may stimulate students in their pursuit of new questions and support them in broadening their mathematics background. The lists also recommend materials that may help teachers in the development of classroom activities that teach the skills of mathematics research or that help students appreciate how doing research makes them part of a grand intellectual enterprise.

If you have arranged to work with a mentor from a college or an experienced colleague, they will be your best resource. You and your students should feel comfortable asking any question about the research experience or admitting that you are not sure what the next question should be. You should also be willing to share your expertise with your mentor. Let her know what your

goals are and what approaches you think might work in your particular teaching setting. Work with her to explore possible activities, questions, and responses that you might use with your students. Be sure to email, fax, or mail copies of as much of your students' work as possible. The more that your mentor can see the type of mathematical thinking that is taking place, the more useful will be the comments that you get back.

Many of us who want to make research a part of our curricula have found that we need to take the time to engage in a research effort ourselves. These investigations help us to enhance our own skills and to appreciate the frustrations and excitement that our students face (see Teacher Affect in Student and Teacher Affect). You can ask your mentor if she will provide advice as you work through a project (either before you begin with your students or at a later date—some teachers have found the summer to be a good time for this work). Find a colleague who would like to join in—make your own research community! Posing new mathematics questions, getting stuck, and thinking about our own thinking as we try to solve a problem is an essential part of our professional development.

Lastly, be sure to take full advantage of the Making Mathematics online materials (the projects and their teaching notes, the Teacher Handbook, and the Mathematics Tools).

BOOKS (see the bibliographies of other teacher handbook chapters as well)

Mathematicians

Albers, Donald and G. Alexanderson (1985). *Mathematical people: profiles and interviews.* Boston, MA: Birkhauser.

Students develop a sense of the growth of the mathematics by studying both the history and present life of the discipline. Reading about mathematicians and the problems that interest them dispels any notion that math is simply a linear sequence from arithmetic to Calculus (see the Problem Posing appendix). These interviews also put varied human faces on a stereotyped field. The interviews in Mathematical People (the one with Persi Diaconis is a favorite of mine) make for effective homework assignments. Students are impressed with the creativity and challenges that the mathematicians face and often curious about the problems discussed. A similar resource is the NOVA Video "The Proof" about Andrew Wiles' proof of Fermat's Last Theorem. A transcript of the interview with Wiles and educator's pages about

this video can be found online at http://www.pbs.org/wgbh/nova/proof. The video can be ordered from Nova Videos (1-800-255-9424).

Books on the history of mathematics (your choice).

Students often think that the subject is complete. Share with them stories of mathematicians creating new mathematics (see the series What's Happening in the Mathematical Sciences by Barry Cipra below) and encourage them to see that they are continuing the process of extending the discipline themselves.

Cipra, Barry. *What's happening in the mathematical sciences, volumes 1, 2, 3, and 4* USA: American Mathematical Society.

Four interesting collections of accessible essays on recent research results, many with applications to other disciplines, written for the lay reader.

The Research Process

Becker, Jerry and Shigeru Shimada, Eds. (1997). *The open-ended approach: a new proposal for teaching mathematics.* Reston, Virginia: NCTM.

This is a translation of a wonderful Japanese book with essays on problem-creation and lesson-planning with the goal of developing higher order thinking skills.

Davis, Philip and Reuben Hersh (1981). *The mathematical experience*. Boston, Massachusetts: Houghton Mifflin Company.

An extensive collection of essays on the nature, process, and philosophy of mathematics and on important mathematical results. Many of the essays make good reading assignments for a research course.

Gerver, Robert (1997). *Writing math research papers*. Berkeley, CA: Key Curriculum Press.

A good general guide about doing mathematics research. It is written so that high school students themselves can use it. It also includes advice for teachers developing a research course. The chapters address not only writing mathematics but also finding questions, proof, and oral presentations.

Mason, John with Leone Burton and Kaye Stacey (1985). *Thinking Mathematically* (*revised edition*). Harlow, England: Addison-Wesley.

This book provides activities and lots of advice about working on problems. It could be used as a text for a research class.

Research Content

Books by Martin Gardner.

Gardner's collection of essays are stimulating introductions to countless important and recreational areas of mathematics. The problems are accessible to students while offering extensions that range from reasonable to still unsolved.

COMAP (1997). *For all practical purposes, 4th ed.* New York, New York: W. H. Freeman.

An engaging textbook on a wide range of mathematical topics (e.g., graph theory, non-Euclidean geometry) and their applications. Many of these topics can be the starting point for additional explorations (both pure and applied).

Stephenson, Frederick (1992). *Exploratory problems in mathematics*. Reston Virginia: NCTM.

Introductory essays on doing mathematics investigations and 60 project ideas with multiple questions and hints for each.

References

Borowski, E. J. and J. M. Borwein (1991). *Mathematics: the HarperCollins dictionary.* New York, New York: HarperPerennial.

A thorough dictionary that works at the high school level. A good mathematics dictionary is particularly helpful when students are reading technical literature.

Wells, David (1986). *The Penguin dictionary of curious and interesting numbers.* London, England: Penguin Books.

A fun resource to flip through. Students will discovery many new ideas in number theory and be stimulated to explore further.

Wells, David (1991). *The Penguin dictionary of curious and interesting geometry.* London, England: Penguin Books.

Same as above only with many beautiful and less well-known geometry results.

PERIODICALS

The College Mathematics Journal (published by the Mathematical Association of America.

Look at back issues for the Student Research Project column, which presents interesting project ideas.

The Mathematical Gazette (UK).

The Gazette publishes articles about the teaching and learning of mathematics, with a focus on the 15-20 age range, and expositions of attractive areas of mathematics. Regular sections include letters, extensive book reviews and a problem corner.

The NCTM magazines Mathematics Teacher and Mathematics Teaching in the Middle School frequently have problems and articles that make for good research investigations.

WEB SITES

Cut the Knot at http://www.cut-the-knot.com has information on countless mathematics topics. A fun site for students to explore. The site includes an excellent list of links to other math sites at http://www.cut-the-knot.com/collection.html.

The Math Forum at http://mathforum.org/ is the pre-eminent K-12 mathematics web site with a search engine for finding out about most mathematics topics of interest.

The Math Archives at http://archives.math.utk.edu is an extensive resource with lesson plans, software, links, and information about mathematics topics.

Meaningfulmath at www.meaningfulmath.org. If you and your students want to pursue research into an applied mathematics topic, this site provides curricular materials for the teaching of mathematical modeling. Lesson plans, handouts, software, and pedagogical advice are provided.

The Geometry Junkyard at http://www.ics.uci.edu/~eppstein/junkyard/. A collection of hundreds of accessible problems. See http://www.ics.uci.edu/~eppstein/ for a broader range of topics and many good links.

Favorite Mathematical Constants at http://pauillac.inria.fr/algo/bsolve/constant/constant.html is a list of constants associated with a range of mathematical problems. The site is fun to explore and can inspire new research ideas.

Problems With A Point at http://www2.edc.org/mathproblems is a searchable database of several hundred problem sets designed to help secondary students build their mathematical content by making connections with already learned ideas. The problem sets also develop the mathematical habits of mind necessary to problem solving and research.

MAA Online - Columns at http://www.maa.org/news/columns.html is a collection of interesting mathematics essays that will introduce students to new areas and applications of

mathematics. For example, students looking for new project ideas might be inspired by Ivars Peterson's essays at http://www.maa.org/mathland/mathland_archives.html.

Eric Weisstein's World of Mathematics at http://mathworld.wolfram.com. An impressive encyclopedia with over 10,000 definitions and descriptions. Another fun site to explore either randomly or with purpose.

Sloane's On-line Encyclopedia of Integer Sequences at http://www.research.att.com:80/~njas/sequences/ is a helpful tool when research produces a numeric pattern that a student cannot identify.

The MacTutor Math History Archive at http://www-groups.dcs.st-and.ac.uk/~history/index.html has over 1000 bibliographies of mathematicians (http://www-groups.dcs.st-and.ac.uk/~history/BiogIndex.html), a collection of essays on historical topics in mathematics (http://www-groups.dcs.st-and.ac.uk/~history/Indexes/HistoryTopics.html), The Famous Curves Index (http://www-groups.dcs.st-and.ac.uk/~history/Curves/Curves.html), The Famous Curves Applet Index of interactive graphs (http://www-groups.dcs.st-and.ac.uk/~history/Java/index.html), and even Maps of Mathematicians' Birthplaces (http://www-groups.dcs.st-and.ac.uk/~history/BirthplaceMaps/MapIndex.html).

Biographies of Women Mathematicians at http://www.agnesscott.edu/lriddle/women/women.htm.

Academic Press Dictionary of Science and Technology: Mathematics & Computers at http://www.harcourt.com/dictionary/browse/math.html. See, in particular, the mathematics information at http://www.harcourt.com/dictionary/browse/19/ and the statistics entries at http://www.harcourt.com/dictionary/browse/76/.

Math Pages at http://www.mathpages.com/ is an organized reference on a range of sophisticated mathematics problems.

Joseph Malkevitch's list of Mathematics Research Projects at http://www.york.cuny.edu/~wu/jm/.

Paper Pool Game at http://Illuminations.nctm.org/imath/6-8/pooltable/pool1.html. An introductory investigation with teacher notes, follow-up questions, and tips on organizing the investigation.

The Dynamical Systems and Technology Project at http://math.bu.edu/DYSYS/. A thorough resource for your students who are curious about fractals and chaos.

Math Book Collection at http://cdl.library.cornell.edu/cdl-math-browse.html. A collection of scanned mathematics books from the nineteenth and early twentieth centuries.

Coping With Math Anxiety at http://www.mathacademy.com/pr/minitext/anxiety/index.asp. An introduction to the issues associated with math anxiety.

Mathematrix at http://www.geocities.com/CapeCanaveral/Hangar/7773/index.html. Recreational mathematics topics and quotes.

Mudd Math Fun Facts at http://www.math.hmc.edu/funfacts/. Quick snippets designed to pique your students' curiosities.

Mathematics Pronunciation Guide at http://waukesha.uwc.edu/mat/kkromare/main.html. Try this site if you want help pronouncing unfamiliar mathematics terms.

INFORMATION LETTER

If you are going to make research a part of a course, an entire course, or an extracurricular activity, you may want to let parents know about your plans and how their children will benefit from them. You may also need to explain the same to school administrators. Feel free to use some, or all, of the following sample letter and to modify the content to your circumstances (send your letters to dmrs@edc.org and we will share them with other teachers):

Dear Parent,

This year our class will engage in a long-term mathematics research experience. This research will provide your child with the opportunity to apply the mathematics skills that s/he has learned to challenging mathematics problems. The students will also get to pose their own questions for investigation, to make conjectures, and to prove their own claims. Student should understand how the discipline works and where mathematical knowledge comes from. Our research strand will help them see mathematics as a broad, deep, and growing body of knowledge to which they can contribute. Such work will involve ample practice with the technical skills of algebra and generate an appreciation for the usefulness of those skills.

We will be using curriculum developed by the *Making Mathematics* project, which supports middle and high school students and teachers who are working on research projects in mathematics. The *Making Mathematics* Web site, http://www2.edc.org/makingmath, and materials were designed by staff of the Center for Mathematics Education at the Education Development Center, Inc. (EDC) with funding by the National Science Foundation. EDC is a non-profit organization based in Newton, Massachusetts.

In addition, Professor Smith of the Department of Mathematics Education at Local University has agreed to read students' reports and provide advice. We are excited to have Professor Smith as a mentor.

Please contact me if you have any questions. I look forward to your child's discoveries.

Sincerely,

