

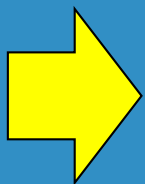
FOUR POWERFUL PRACTICES FOR TEACHING MATH INTERVENTION IN THE MIDDLE GRADES

August 30, 2022

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Education Development Center

Welcome!



Please introduce yourself in the CHAT: name, role, and location.

EDC

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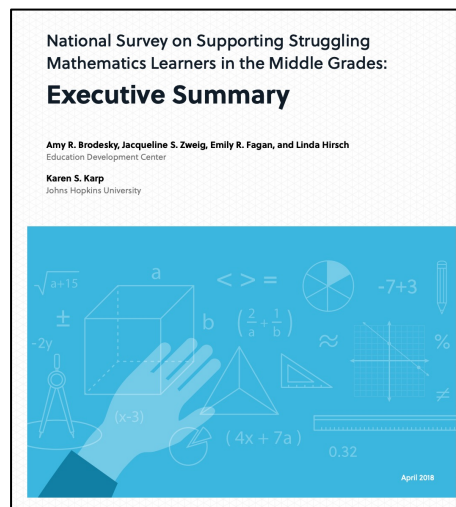


How have we studied math intervention at the middle grades?



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- **Review of Relevant Research**
- **Observations of Math Intervention Classes**
- **Interviews with Teachers and Leaders**
- **National Survey of 876 public schools (Grades 6-8)**
- **Professional Development Courses for**



Agenda



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- Introduction
- 4 Powerful Instructional Practices
- Time for Questions

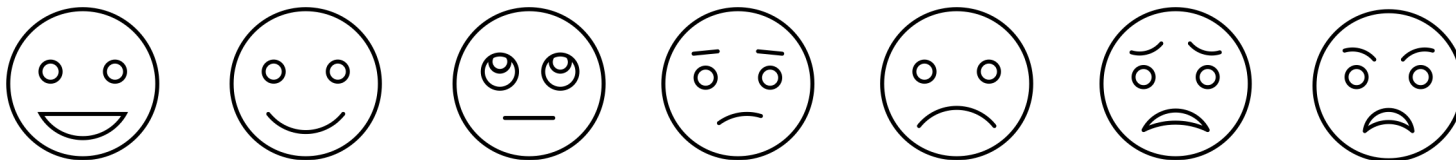
Please type questions in the Chat.

Middle grades students may need math intervention for different reasons:



- Prior difficulties with elementary math
- Content gets more abstract and pace increases
- Non-academic factors and stressors
- Learning differences and disabilities
- Interrupted learning due to pandemic
- Missing content due to changing districts

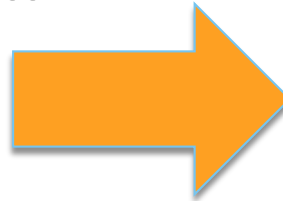
Students may have a mix of feelings about math intervention:



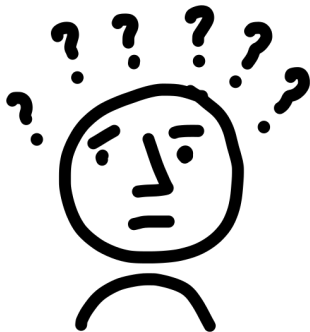
Math Intervention can help support students in making important shifts

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- Feel anxious about doing math
- Lack motivation
- Lack self-confidence
- Display learned helplessness
- Hide their difficulties
- Avoid doing math
- Feel overwhelmed



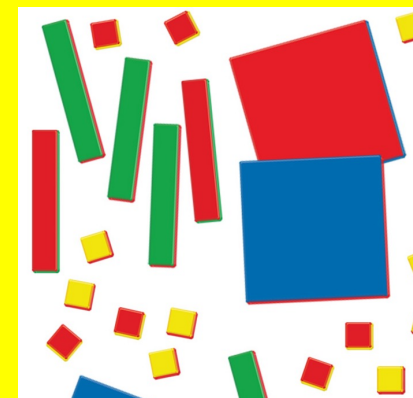
- Feel Comfortable & Safe
- Gain Confidence
- Increase Motivation
- Engage, participate
- Ask questions
- Share thinking
- Feel empowered





Practices for Math Intervention Class

1. Use Strategies to Make the Most of Concrete Representations



Research Highlights:

Using Concrete & Semi-Concrete Representations

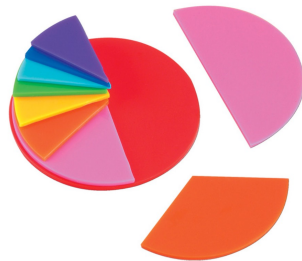
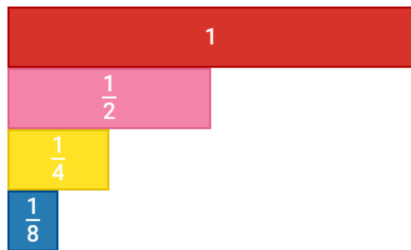
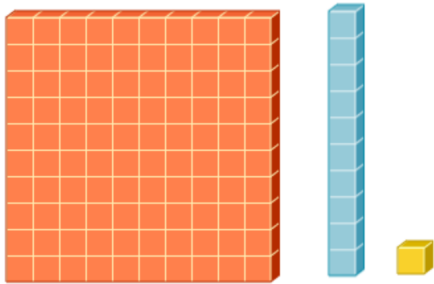
- Use as thinking tools for making sense and solving math problems
- Use as communication tools
- Connect concrete, semi-concrete, and abstract representations
- Provide ample and meaningful opportunities – Don't rush to fade
- Convey positive messages about value of representations throughout the grades
- Carefully choose representations

Put in chat: What is a favorite concrete or visual representation? Why?

Use Proportional Manipulatives for Fractions and Decimals

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Pieces are Proportional



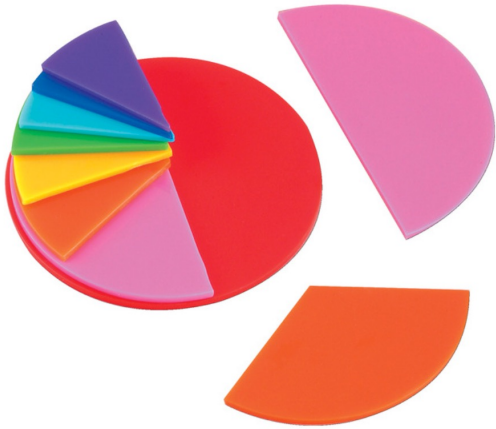
Pieces are Not Proportional



Proportional relationship of the pieces brings out the math concepts

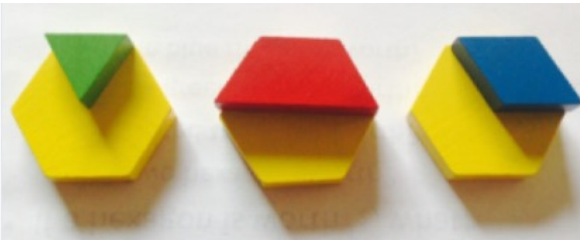
Choose Models to Support Different Math Ideas:

Area Models



Fraction Circles: Benefits

- See what a numeric fraction means in terms of piece size, number of pieces, equal-size pieces
- Explore ways to make a whole
- Build mental images of fractions



Pattern Blocks: Benefits

- Pieces have color and shape attributes
- Explore changing the whole
- Brings out need to attend to what the whole is

Linear Model

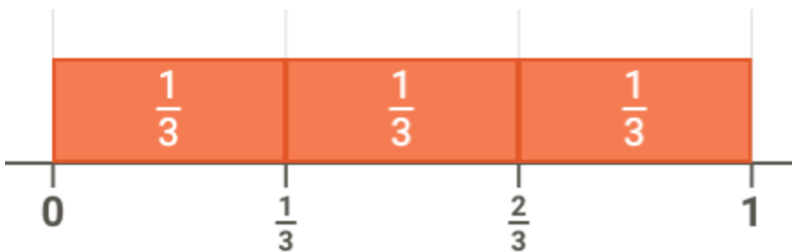
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Fraction bars/tiles



Benefits:

- See what a numeric fraction means in terms of piece size, number pieces, equal-size
- Explore ways to make a whole
- Model fractions and sums greater than 1
- Model subtraction and division
- Model multiplication: whole number \times fraction
- Connects to number line representation



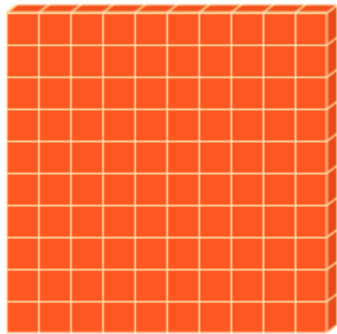
Connect Manipulatives to Math Concepts

Be careful not to assume that students will see how a manipulative represents a math concept in the way that we do as adults

Example of a student's non-mathematical explanation

Why is the value of a rod one tenth?

Teacher



One



One-Tenth

"I know it's a tenth because it's blue."

Student

What's missing from the student's explanation?

Connect Representations: Use Vertical Alignment and Color

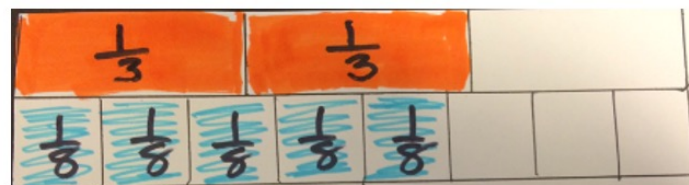


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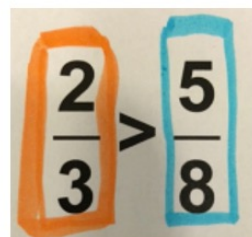
Concrete



Semi-Concrete



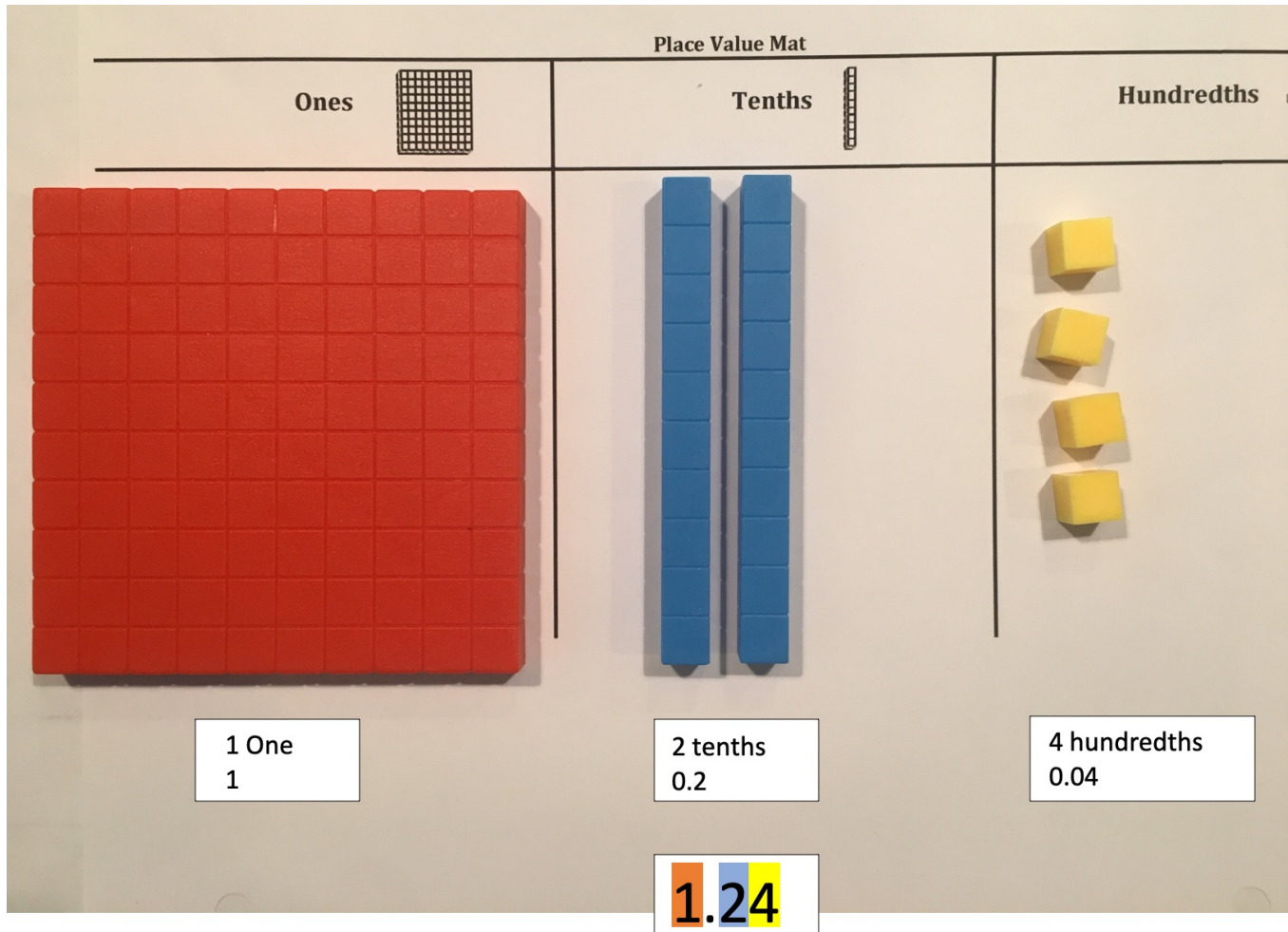
Abstract



Connect Representations: Use Organizers, Mats, and Color



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Use Questions to Bring Out the Math Ideas:

Ask student to...

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Connect Parts of Representations

- Where is the [____] in each representation?
- How do the parts of the [representation] connect to the [equation]?

Describe Strategies

- What actions did you use with the [manips] to find the sum? Show what you did.
- How did you use your drawing to represent the fractions and find the sum?

Use Reasoning and Justify Answers

- Is the sum of the two fractions greater than or less than 1 whole? How do you know? Use your [manipulatives] to explain your answer.

Concrete Representations: Not Just Manipulatives!

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- Kinesthetic Activities
 - ▣ Create clothesline number lines for fractions
 - ▣ Walk on the number line or coordinate grid
 - ▣ Holding up cards with decimals and lining up in order

- Act Out Problem Situations & Use Actual Objects



Practices for Math Intervention Class

2. Support Students in Making and Using Visuals Productively

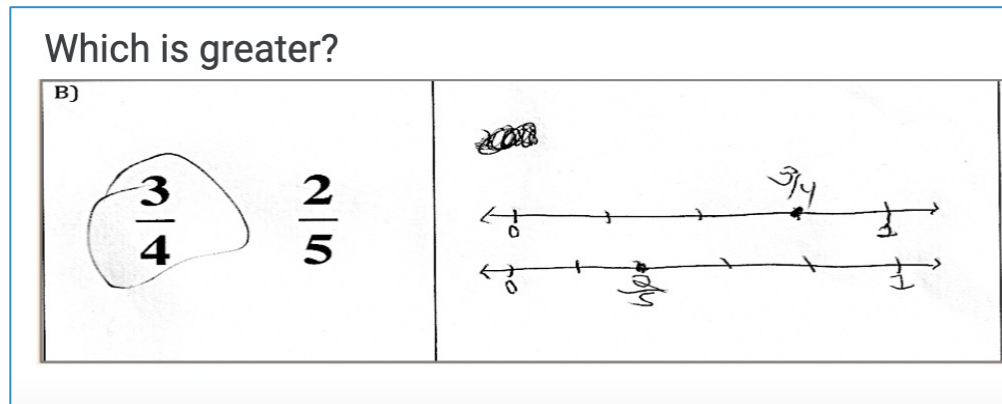
Explain your choice using words and/or pictures

4 are bigger pieces

$\frac{1}{10}$ is smaller pieces

Research Highlights: Number Lines

- Use number lines to build understanding of fractions, decimals, and integers



- Use a variety of number lines with different partitions and endpoints – not just 0-1

Research Highlights: Visual Representations and Drawings

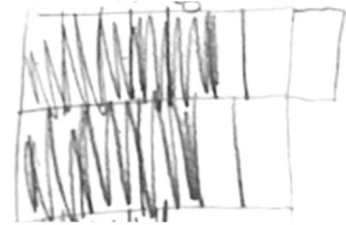
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- Making and using visual representations is helpful student math learning
- Positively correlated with higher performance on word problems
- Students with learning disabilities tended to have a weaker understandings of what a diagram is
- Students who have math difficulties tend to have challenges with making drawings and may need instruction on how to make useful drawings for representing/solving problems

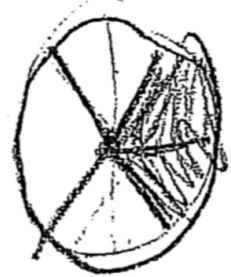
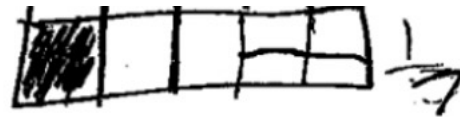
Example Difficulties with Drawing Fractions

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A. Draw wholes that are different sizes



B. Partition whole incorrectly; do not create equal parts



$\frac{2}{5}$

C. Shade the parts so darkly that they can no longer see the pieces



Common Student Challenges for Making Drawings

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- A. Unsure what to draw to represent a math situation
- B. Unsure how to use a drawing to solve a problem
- C. Spend too much time on making drawings, draw slowly or use too much detail
- D. Focus on drawing the superficial aspects of a problem
- E. Lack the precision needed to solve a problem
- F. Leave out important information (from problem) in their drawing
- G. Make errors in the drawing, such as showing incorrect quantities

Suggestions: Support Students with Making and Using Drawings



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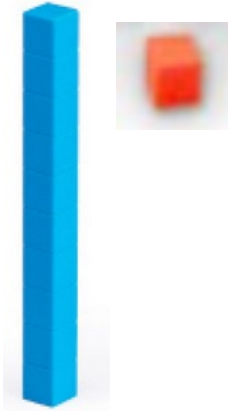
- A. Provide instruction on making drawings for specific math concepts
- B. Clarify expectations for a “simple drawing” or “sketch”
- C. Clarify the amount of precision needed for a specific task
- D. Model making drawings by recording students’ work w/manips.
- E. Use labeling and arrows to connect visual and numeric
- F. Be strategic in choosing when to have students make drawings
- G. Adapt problems to make them easier for students to draw

Put in the Chat: What’s one strategy that you use?

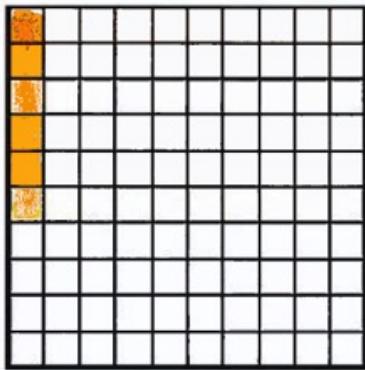
Use Color Strategically to Connect Visual Representations

22

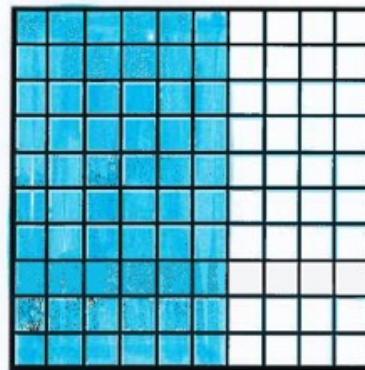
Example: Compare 0.06 to 0.6 using grids and number line



0.06

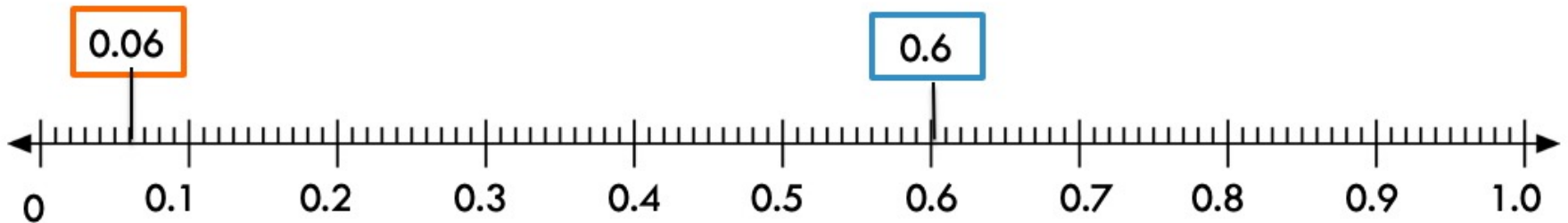


0.6

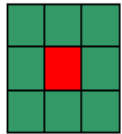


0.06

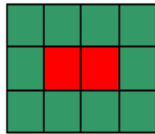
0.6



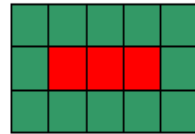
Link Sheet and Strategic Use of Color



Garden 1

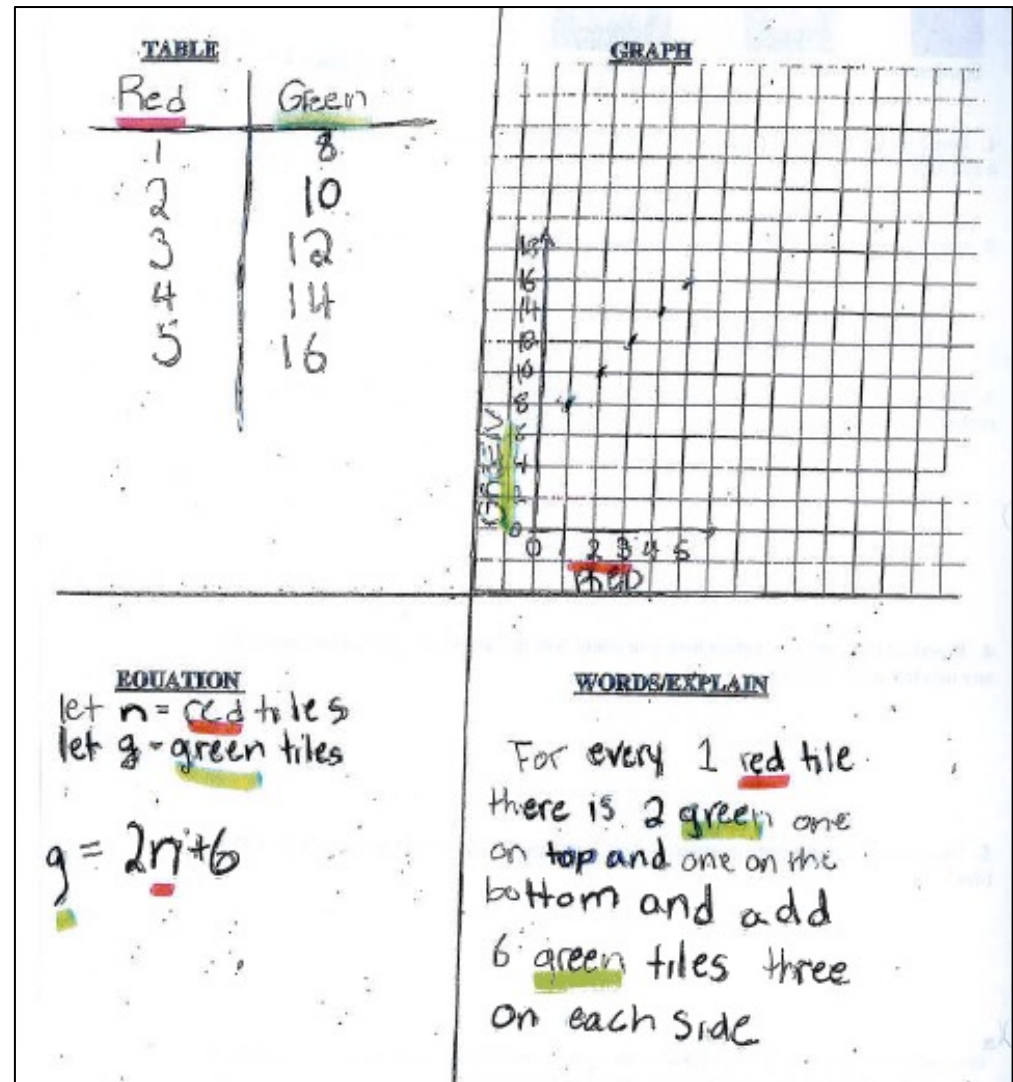


Garden 2



Garden 3

- Draw Garden 4 and Garden 5.
- How many green tiles do you need for Garden 4? Garden 5?
- How can you find the number of green tiles needed for any garden?





Practices for Math Intervention Class

3. Promote Student Communication

Communication is important



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Struggling learners may be hesitant to talk in class.

When students share their ideas, ...

- they solidify understandings
- they find mistakes and correct them
- teachers gain insight into their thinking
- It helps build a learning community

Sentence Starters or Stems

- First, I did...
- I agree because...
- I know this makes sense because...

Sentence Frames

- I chose the fraction _____ because it is _____ than $\frac{1}{2}$.

Sentence Frame Example: Which One Doesn't Belong?

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| | |
|----------------|-----------------|
| $\frac{1}{20}$ | $\frac{20}{25}$ |
| $\frac{2}{3}$ | $\frac{5}{4}$ |

Source: <http://wodb.ca/index.html>

Support more math-focused communication:

- ▣ “I noticed that _____ doesn’t belong because _____”
- ▣ Use rewording and elaboration to bring out key math ideas and highlight vocabulary

Unit
fraction

Non-unit
fraction

Numerator

Denominator

- ▣ Partner conversation and whole group share out

Strategies: Mini-Whiteboards and “Show me an example...”

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Show me an example of...

- Two numbers that have a sum of -4



More Show-me Examples

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- A shape with a small area and a large perimeter
- Two fractions that have a sum greater than 1
- A set of 5 numbers with a range of 6... and a mean of 10



“Show Me an example of...”

Benefits

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- Teachers can see what each student did
- Students can show responses to the teacher and to each other
- Having a visual to show can help students to talk about their ideas and approach
- Helps make the discussion more concrete
- Students can easily erase and change their work on the whiteboard – “rough draft”

Using engaging math activities in math intervention is essential, but not sufficient, for promoting **student communication.**
What supports are helpful?

Use Games with Communication Supports

Aim-HIGH

| | | | | |
|--|--|---|--|--|
| | | . | | |
|--|--|---|--|--|

Aim-LOW

| | | | | |
|--|--|---|--|--|
| | | . | | |
|--|--|---|--|--|



Sentence Frame

“I picked _____. I am putting _____ in the _____ place of my _____ number because...”

Example: I picked 7. I'm placing 7 in the **hundredths** place of my **Aim-LOW** number because...

Suggestions

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- Choose activities that
 - ▣ Have accessible entry point: *all students will be able to have something to say*
 - ▣ Allow multiple approaches/ideas: *students will have different things to say*
- Set expectations: “I’m going to listen for...”
- Use protocols and roles to support conversations
- Have students present with a partner

Avoid:

- Teaching vocabulary in isolation
- Not planning enough time for students to share and explain their work



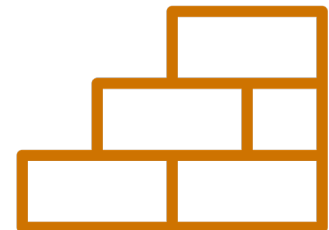
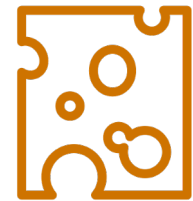
Practices for Math Intervention Class

4. Use Formative Assessment to Identify Students' Strengths and Needs

“My students have many gaps. It’s too much content to teach in the time we have. I’m not sure what to focus on.”

Consider reframing from ‘filling gaps’ to ‘building a strong foundation’ to support student success

Identify and build on strengths



Students in Math Intervention may...

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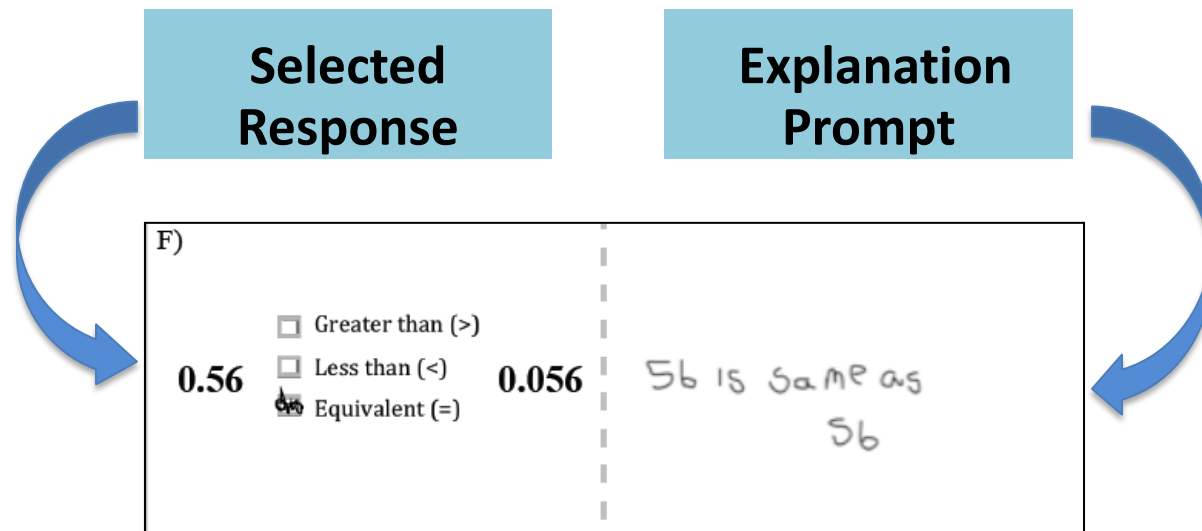
- ▣ **Feel anxiety** about math tests and assessments
- ▣ Have **more understanding** than what is reflected in their written work and assessment performance
- ▣ Hold **common misconceptions** that are impeding their progress.

How can we find out about students' understandings, difficulties, and possible misconceptions in ways that are supportive and efficient?

Formative Assessment Probes

Focused collection of diagnostic items that targets research-based misconceptions

Format allows for quick administration (about 10 min)



Probe: Comparing Decimals

| Choose the correct response | Explain your thinking: |
|---|------------------------|
| <p>A)</p> <p><input type="checkbox"/> Greater than (>)</p> <p>0.175 <input type="checkbox"/> Less than (<) 0.2</p> <p><input type="checkbox"/> Equivalent (=)</p> | |
| <p>B)</p> <p><input type="checkbox"/> Greater than (>)</p> <p>0.31 <input type="checkbox"/> Less than (<) 0.426</p> <p><input type="checkbox"/> Equivalent (=)</p> | |
| <p>C)</p> <p><input type="checkbox"/> Greater than (>)</p> <p>0.25 <input type="checkbox"/> Less than (<) 0.0756</p> <p><input type="checkbox"/> Equivalent (=)</p> | |

Probe: Comparing Decimals

Student Work Examples

- 0.175** Greater than ($>$) **0.2**
 Less than ($<$)
 Equivalent ($=$)

0.2 has less numbers than 0.175 so it must be less than

D)

- 2.3** Greater than ($>$) **2.30**
 Less than ($<$)
 Equivalent ($=$)
- tenths hundredths

tenths ~~are~~ larger than hundredths

Probe: Comparing Fractions

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| | |
|--|--|
| <p>2.</p> <p>$\frac{6}{8}$ <input type="checkbox"/> Greater than (>) $\frac{3}{4}$ <input type="checkbox"/> Less than (<) $\frac{3}{4}$ <input type="checkbox"/> Equivalent (=) $\frac{3}{4}$</p> | <p>Explain your choice using words and/or pictures</p> |
| <p>3.</p> <p>$\frac{4}{5}$ <input type="checkbox"/> Greater than (>) $\frac{6}{7}$ <input type="checkbox"/> Less than (<) $\frac{6}{7}$ <input type="checkbox"/> Equivalent (=) $\frac{6}{7}$</p> | <p>Explain your choice using words and/or pictures</p> |
| <p>4.</p> <p>$\frac{5}{12}$ <input type="checkbox"/> Greater than (>) $\frac{3}{4}$ <input type="checkbox"/> Less than (<) $\frac{3}{4}$ <input type="checkbox"/> Equivalent (=) $\frac{3}{4}$</p> | <p>Explain your choice using words and/or pictures</p> |

Probe: Comparing Fractions

Student Work Examples

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| | |
|--|---|
| <p>2.</p> <p><input checked="" type="radio"/> Greater than ($>$)</p> <p><input type="radio"/> Less than ($<$)</p> <p><input type="radio"/> Equivalent ($=$)</p> <p>$\frac{4}{5}$ $\frac{6}{7}$</p> | <p>Explain your choice using words and/or pictures.</p> <p>The smaller the bottom number the bigger the peices.</p> |
|--|---|

| | |
|---|---|
| <p>4.</p> <p><input checked="" type="radio"/> Greater than ($>$)</p> <p><input type="radio"/> Less than ($<$)</p> <p><input type="radio"/> Equivalent ($=$)</p> <p>$\frac{5}{12}$ $\frac{3}{4}$</p> | <p>Explain your choice using words and/or pictures</p> <p>$\frac{5}{12}$ $\frac{3}{4}$</p> <p>12 is much bigger than 4</p> |
|---|---|

Variety of Ways to Administer Probes

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- Paper/Pencil
- Student Interview
- Card Sorts

Equivalent to $4x + 12y$

Not
Equivalent to $4x + 12y$

A

$$2(2x + 6y)$$

D

$$8x + 24y$$

E

$$(4 + 12)(x + y)$$

- Link to probes on the Resources document

Planning Targeted Instruction

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- Use findings to set learning goals
 - ▣ Continue with the math content assessed by the probe?
 - ▣ Move back to prerequisite foundational content?
 - ▣ Move ahead to new math content?
- Plan focused instruction to support goals and students' needs
 - ▣ Choose activities to build on strengths and address difficulties

Summary:

Four Powerful Practices for Math Intervention



1. Use Strategies to Make the Most of Concrete Representations
2. Support Students in Making and Using Visuals Productively
3. Promote Student Communication
4. Use Formative Assessment to Identify Students' Strengths and Needs

Guiding Principles for Math Intervention



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In today's session, we focused on the highlighted principles:

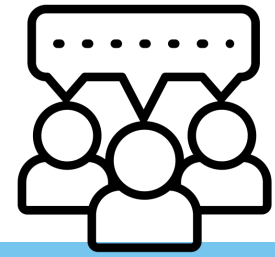
1. Engage students in doing meaningful mathematics to build conceptual understanding and sense-making
2. Identify content priorities
3. Uncover and assess
4. Target instruction
5. Set and communicate clear learning goals
6. Promote student communication
7. Empower active, independent math learners
8. Build a learning community
9. Celebrate success
10. Collaborate with colleagues



YOUR QUESTIONS



Ideas to Apply



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What's one idea from today's session that you want to apply in your practice?

Share an idea in the chat.

Contact Information

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