TOOLKIT

Middle Grades Mathematics Instruction for Multilingual Learners

Strategies for Success

EDC Education Development Center

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Background

Go to Toolkit Overview

Language is deeply involved in the learning of mathematics. English proficiency, however, is *not* a prerequisite for doing mathematical work or for engaging in reasoning and problem solving. Classroom strategies that help learners more easily access and use mathematical language and promote problem solving and participation can support the integration of academic English into mathematical work. Multilingual learners benefit from regular participation in explaining their thinking, describing their solutions, and presenting their mathematical ideas. Language-rich environments provide multilingual learners with examples of mathematical discourse and opportunities to participate in it.

This toolkit offers strategies for teaching multilingual learners in mathematics and specific steps for classroom implementation. The strategies are organized into three categories:

1. Setting Up Students for Success
2. Facilitating Problem Solving Through Diagramming
3. Promoting Participation in Mathematical Discourse

> **"These strategies were really valuable for my students. They encouraged them to have conversations with each other about mathematics and about their own thinking about it. They provide a way for all students to engage in the mathematics."**

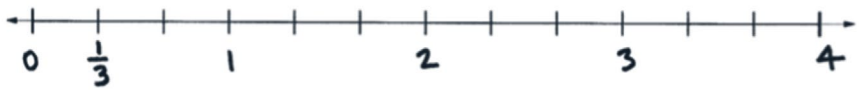
Toolkit Overview

"ELs bring multicompetence to the STEM classroom, with broader aspects of language knowledge and cultural knowledge than monolingual speakers"

- National Academies of Sciences, Engineering, and Medicine (2018)

This toolkit offers several classroom-tested strategies to facilitate the mathematical thinking and learning of multilingual learners by drawing on their strengths. We use the term *multilingual learners* to refer to students identified by schools as English learners in order to highlight the strengths that students bring to the classroom.

Click on a topic box to learn more.



Setting Up Students for Success

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Introduction | Providing Access to Language | Three Reads Strategy | Acting Out Tasks

Students' active engagement in mathematical problem solving and discussion is critical to their learning. To get multilingual learners started in their problem solving and discussion, strategies to support students in reading mathematics tasks and communicating their ideas are important. The instructional strategies described in this section can facilitate multilingual learners' active engagement and reduce some linguistic demands inherent in communicating about mathematics. These strategies, however, must be embedded *in the context of mathematical work* and not taught separately from the mathematics or without any connection to a task or activity. Multilingual learners develop their understanding of content and language concurrently: Academic content serves as the context for language learning, and language facilitates the learning of academic content (WIDA, 2020). This means that language support strategies need to be integrated with the mathematics task, activity, or problem on hand and not separated from the mathematics (Moschkovich, 1999).

The three research-based instructional strategies described in this section—**Providing Access to Language, Three Reads Strategy,** and **Acting Out Tasks**—help all learners access the language involved in mathematics tasks and lessons and in particular support multilingual learners.

> **"Setting my students up for success requires knowing them and their language strengths, framing the context for the lesson, and making sure they engage with the language."**

Setting Up Students for Success

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A natural inclination is to want to pre-teach students all the vocabulary they will encounter; however, this type of pre-teaching can actually be counterproductive. There are many different types of mathematical talk and language, and pre-teaching vocabulary can both lower the mathematical rigor of a task and take time away from students' engagement in mathematics. Integrating language instruction into the mathematical work is key so that students build their understanding of terms and phrases *in context*.

Multilingual learners need opportunities to respond to questions and communicate mathematically about ideas, arguments, and conclusions, using both academic and non-academic vocabulary (WIDA, 2020). They need to learn and practice academic language functions in context, such as analyzing, describing, and evaluating (Heinek & Neugebar, 2018), and to learn both common and discipline-specific meanings of words and terms, such as *property and base* (Driscoll et al., 2016).

> **"It's really important to support my multilingual learners by creating shared contexts. We can do that by finding ways to demonstrate information. Favorites for us were acting out the problem, and using props or drawings. These gave students opportunities to make meaning before working to use precise vocabulary. We allow them to think and understand before asking them to use language that is unfamiliar."**

Continued

Tips for providing access to language:

- To decide what terms or phrases may need clarification at some point, review the language used in tasks across the lesson or unit. Some vocabulary, terms, and phrases—and in particular their meaning in *mathematics*—may be unfamiliar to students. Think carefully about which terms need clarification and which do not, and when and how during the lesson these terms should be clarified.
- To better understand how students are making sense of the language and what terms or phrases may need clarification, use a co-constructed word bank where you and the students each contribute any words and phrases from a particular task that are important or confusing. Using color or another visual cue to group related terms as they are added to the word bank can further support students in building meaning about those terms.
- Aim to provide students with just enough access to the language of a task so that they can work toward the mathematics and language objectives of the lesson. Defining terms should not replace students' involvement in the mathematical work but instead should help students understand definitions to strengthen their understanding of the task and their ability to communicate about it.
- You may clarify key terms at different points in the lesson, depending on the accessibility of the language, students' prior in-class learning, and the mathematics task itself.
- When clarifying vocabulary, encourage students to first share their own understanding of the term. You can then build on their understanding to include mathematical meanings that are specific to the task.

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The Three Reads strategy facilitates sense-making of word problems and boosts access for multilingual learners (Driscoll et al., 2016). This strategy involves reading a mathematics task three times, with a different purpose for each read:

1. **The first read is to understand the "story" or context of the task.** Students should not focus on specific quantities or the relationships between them during this reading. Responses should be more like a title for the problem, such as "Ice cream," "Sharing ice cream," or "Flowerpots and soil." Students complete the sentence starter, "The problem is about . . ."
2. **The second read is to discern the question posed by or the goal of the task.** Students complete the sentence starter, "I need to find out . . ."
3. **The third read is to gather the important information needed to work on the problem**, such as specific quantities and their relationships. Students complete the sentence starter, "The important information is . . ."

Use the **Three Reads Strategy** for the Filling Flower Pots task:

1st Read – Context
The problem is about...

2nd Read – Purpose
I need to find out...

3rd Read – Information
The important information is...

Continued

Setting Up Students for Success

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Reading a mathematics word problem requires a different approach from reading prose, in part because the purpose or question typically does not appear until the end of the passage. Since the reader does not know the purpose until the end, it can be difficult to determine the importance of the information within the text. In addition, the text of a mathematics problem is often dense and may include academic language, complex sentences, or unfamiliar ways to express relationships. Reading it more than once may strengthen students' understanding. A focus on the mathematical context, rather than the specific vocabulary, grammar, or sentence structure, supports students in building on the mathematical skills and concepts they already know.

> **"The Three Reads strategy helps me get started on a task because it makes it easier for me to solve the problem."** - Grade 6 student

Tips for using the Three Reads strategy:

- ☑ Although this strategy can be successfully used individually, repeated reading out loud in a small group or as a class provides opportunities for students to both hear and see the language multiple times.
- ☑ After each read, provide students with brief think time and then have multiple students share their responses. Having several students share with the whole group allows all students to hear the information and compare it with their own ideas. It also promotes connections between different ways of phrasing the ideas.
- ☑ During the second read, encourage students to focus on the purpose of their work on the problem (such as, "Find the number of servings") rather than on mathematical operations or next steps (such as, "I need to divide").
- ☑ Record students' responses. After the third read, indicate which information that students identified is correct or important to keep in mind before moving on to work on the task.
- ☑ Remind students to reference their Three Reads strategy notes during their later work on the task. For example, if they're asked to draw a diagram, prompt students to represent the important information they identified during the Three Reads strategy.

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Acting out mathematics tasks supports multilingual learners in interpreting text because they can simultaneously watch an enactment of the task context and hear the language in the task (Driscoll et al., 2016). In the example to the right, by acting out pouring soil into flower pots in a problem about how much soil is needed per flower pot, students can develop an understanding of the problem context. Seeing and experiencing real objects (in this case, flower pots, bags, and soil) can help students interpret the problem context and learn new vocabulary.

Example

The table to the right presents two tasks with similar real-world contexts but different mathematical givens and questions. The considerations for acting out each task are different, as described on the next page.

Task and important information	Task objective
TASK 1: Grandma has 4 bags of soil for her flower pots. Each flower pot needs 3/4 of a bag of soil. How many flower pots can she fill?	**KNOWN:** Total amount of soil, amount of soil needed for each pot **UNKNOWN:** How many pots can be filled?
TASK 2: 5 bags of soil fill 4 red flower pots. The red flower pots are all the same size. How much soil is in each red flower pot?	**KNOWN:** Number of flower pots, total amount of soil **UNKNOWN:** How much soil goes in each pot?

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In Task 1, the objective is to determine how many flower pots can be filled with a set amount of soil. In Task 2, the objective is to determine the amount of soil that goes in each pot for a set number of pots. Because the task objectives, what is known, and what is unknown are different across the tasks, the tasks will be acted out differently:

For Task 1, you could use four small bags of soil and five or six flower pots. To act out the task, begin pouring three-quarters of a bag of soil into one or two pots before pausing so the problem solvers can puzzle through how many total flower pots can be filled.

For Task 2, you could use all four flower pots and five small bags of soil. To act out the task, pour one bag of soil across all four flower pots before pausing so the problem solvers can puzzle through how much soil will be in each pot.

Tips for using the Acting Out strategy:

- ☑ Plan in advance which part of the task makes sense to act out and what information you want to highlight, based on the mathematical context and questions.
- ☑ If also using the Three Reads strategy, integrate acting out the task with the first or second read.
- ☑ You may use actual materials from a word problem when acting it out (such as, flower pot and bag of real soil), substitute a similar material (such as, sand instead of soil), or pretend to use a material (such as, by labeling an empty bag as soil and pretending to pour from it).
- ☑ If acting out is not possible, use videos or pictures to support students' access to the context.

Facilitating Problem Solving Through Diagramming

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Introduction | Starting with Partial Diagrams | Analyzing Example Diagrams

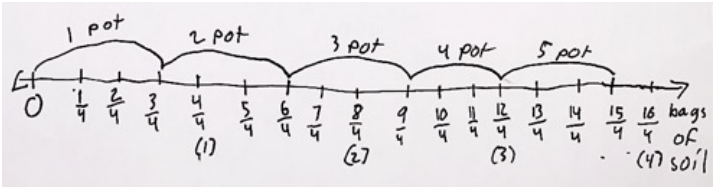
Mathematical diagrams—graphic creations such as number lines and tape diagrams that illustrate relationships among quantities—are not simply a scaffold or crutch for novice problem solvers. When used as thinking tools and not just as tools for presentation, they can facilitate students' mathematical reasoning, problem solving, and communication. Competent mathematical thinkers use diagrams flexibly in problem solving (Stylianou & Silver, 2004).

Teaching students how to use diagrams has a number of benefits for all students, and multiple advantages for multilingual learners. Using diagrams can, for example:

- Help students begin problem-solving by linking quantities and relationships from the problem to the mathematical operations needed for a solution—and ultimately enhance their mathematical problem-solving (Woodward et al., 2012)
- Help students identify and present the important elements needed to solve a task
- Support students in communicating about their thinking
- Widen multilingual learners' access to mathematical thinking and communication, enabling them to participate productively in mathematical discourse by using diagrams to explain relationships between quantities or to interpret the mathematical talk they hear from others

This section describes two strategies that can be helpful for supporting students in their use of mathematical diagrams:

- **Starting with Partial Diagrams**
- **Analyzing Example Diagrams**



> **"Diagrams help my students create a pathway to the solution and then they are able to use that to solve the problem and create connections to algebraic concepts. "**

Facilitating Problem Solving Through Diagramming

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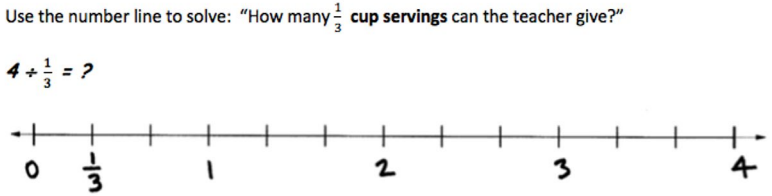
Introduction | Starting with Partial Diagrams | Analyzing Example Diagrams

A *partial diagram* includes some initial components for solving the task at hand, but it is incomplete. Students can work from the partial diagram and add to it in order to complete their problem solving for the task. The partial diagram can support students in finding an entry point for a particular task. More importantly, it can then provide students with ideas for starting points for future problem-solving contexts. Partial diagrams may be embedded in a curriculum, or a teacher can add them to tasks within a curriculum.

Example

In this number line, someone has taken an initial step by partitioning based on a relevant quantity. Students can use this partial diagram as a starting point to solve the problem themselves.



Tips for using partial diagrams to facilitate students' use of diagrams in problem solving:

- ☑ Embed a partial diagram in a mathematics task for all students to encourage students to consider a particular strategy for starting a diagram. Alternatively, provide a partial diagram as an extra resource or scaffold for any students who struggle to get started.
- ☑ Use student-generated partial diagrams to offer entry points for students who are struggling to get started by pausing the class after a few minutes of individual work time and having one or two students share how they started their diagrams.
- ☑ If students are not sure how to add to a partial diagram, prompt them to review the important quantities identified in the task and consider which quantities are already represented and which could be added to the diagram.

Facilitating Problem Solving Through Diagramming

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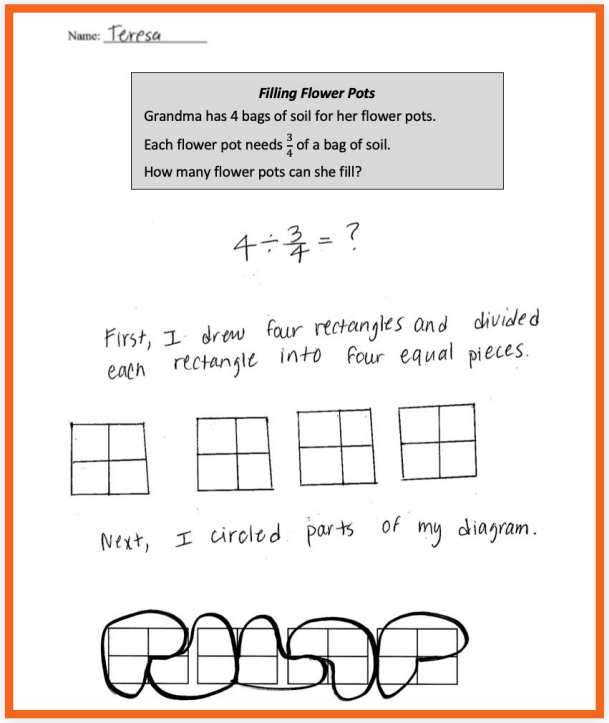
Introduction | Starting with Partial Diagrams | Analyzing Example Diagrams

Analyzing diagrams can help students notice quantities and relationships and can support students in developing flexible thinking in their problem solving. One approach to supporting students' use of diagrams is to have them analyze a worked example—a fictional student's work on a diagram or their problem-solving approach (Booth et al., 2015). A worked example should not be presented as the only solution or the "best way," but rather as one way to represent quantities and relationships in order to solve a problem. The example can either be a correct diagram or approach, or one with a mistake (with clear guidance for students about identifying the error); each provides an opportunity for students to discuss and learn from one person's approach to a problem or task.

Teachers can provide example diagrams for analysis either during problem-solving time or during full-group sharing. The example diagrams can be one the teacher generated or found, or, after students create their own diagrams, the teacher can carefully select one or two examples for analysis that students generated that day. Allowing students to compare different solution strategies for the same task helps them develop more flexibility in their approach to problem solving.

Example

This worked example provides an opportunity for students to analyze Teresa's steps in her problem-solving approach:


Name: Teresa

Filling Flower Pots

Grandma has 4 bags of soil for her flower pots.

Each flower pot needs $\frac{3}{4}$ of a bag of soil.

How many flower pots can she fill?

$4 \div \frac{3}{4} = ?$

First, I drew four rectangles and divided each rectangle into four equal pieces.

Next, I circled parts of my diagram.

Continued

Facilitating Problem Solving Through Diagramming

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Introduction | Starting with Partial Diagrams | Analyzing Example Diagrams

Tips for facilitating students' analysis of diagrams in problem solving:

- ☑ When incorporating worked examples, make sure to consider what you want students to focus on. The example above focuses students on Teresa's problem-solving approach. Alternatively, a worked example could focus on a specific mathematical idea, such as how ¼ of a bag of leftover soil could be ⅓ of what is needed to fill a flower pot.

- ☑ When presenting a worked example, use prompts that facilitate students' thinking and attention to particular ideas or to particular quantities and relationships in a diagram. Possible prompts for the example above are:
  - What does each rectangle represent?
  - Why did Teresa divide each rectangle into four equal pieces?
  - Number the flower pots in Teresa's diagram.
  - Teresa found out that 4 bags of soil can fill ____ flower pots.
  - Shade in the part of Teresa's diagram that shows the soil that is left over after filling flower pots. What fraction of a bag of soil is left over?

- ☑ Include sentence starters with the questions you pose to encourage students to use academic language they may not otherwise use. This approach can also support students of different language proficiency levels in participating in verbal and written mathematical communication about the diagrams.

Promoting Participation in Mathematical Discourse

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Introduction | Sentence Starters and Frames | Partner Work | Full Group Discussion

Multilingual learners have mathematical ideas to share but may struggle to explain their thinking in English. It is important to support these learners in producing and sharing mathematical explanations. Instructional strategies such as those described in this section can reduce some of the linguistic demands inherent in mathematical communication. Students benefit from these strategies in the context of the mathematical work, that is, as strategies that happen during their engagement in a mathematics task.

When facilitating students' participation, make sure to focus on their mathematical strengths in their use of language, diagrams, and calculations. For example, focus on what students do well rather than on their mistakes, and note when a multilingual learner has expressed a new idea that has not come up before in the discussion or has contributed an important mathematical connection. Let your knowledge of your students' mathematical experiences, their language history, and their educational background guide you in supporting them (Moschkovich, 2013).

This section highlights three strategies that can encourage multilingual learners' participation and highlight their mathematical contributions:

- Sentence Starters and Frames
- Partner Work
- Full-Group Discussions

Promoting Participation in Mathematical Discourse

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Introduction | Sentence Starters and Frames | Partner Work | Full Group Discussion

Sentence starters and frames are incomplete sentences for students to complete that provide a structure for students' writing and speaking. Sentence starters provide the beginning of a sentence, and sentence frames provide multiple parts of a sentence with missing parts in the middle. When given a sentence starter or frame, students can spend less time interpreting a question and can instead focus on formulating their ideas.

Sentence starters and frames are useful tools for a number of reasons:

- They provide a starting place for multilingual leaners to begin sharing their explanations and reflections.
- They can support students in sharing their mathematical ideas verbally and in writing, starting with more informal language and progressing to using academic language they may not otherwise use.
- They can elicit students' explanations about their solution steps and encourage mathematical conversations with other students.
- They can support students in using specific academic terms, phrases, constructions, and pairs of terms that go together mathematically.

Example

Here are some examples of sentence starters and frames that you might use in conjunction with student discussion about a diagramming task or diagramming example:

- In her diagram, Teresa divided each rectangle into four equal pieces because . . .
- Diagrams that show . . . can help . . .
- In this diagram, I see . . . In this diagram, I don't see . . .

> **"Those [sentence starters] are neat. I like them, too. They give you words. So you can read to your partner and describe it and stuff, see what you like or not so you can change it. [. . .] The [sentence starters] helped get the conversation started."** -Grade 6 student

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Promoting Participation in Mathematical Discourse

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Introduction | Sentence Starters and Frames | Partner Work | Full Group Discussion

Tips for using sentence starters and frames:

- ☑ Use this strategy to support students in verbally sharing their ideas by crafting sentence starters and frames that have multiple correct answers. Do not use them as a way to evaluate students' thinking.
- ☑ Build sentence starters and frames into the beginning, middle, and end of lessons.
- ☑ Encourage students to think of sentence starters and frames as an opportunity to share their thinking. Remember that students are working to develop an understanding of how to share their ideas, and therefore their completed sentence starters should be considered "rough drafts." It's okay if they misspell words or use informal language.
- ☑ Vary the sentence starters and frames to accommodate a range of English proficiency levels:
  - Ask students with lower English proficiency to add only a word or two to a sentence. This gives them an opportunity to practice using mathematical language as they communicate their mathematical ideas.
  - Ask students with higher English proficiency levels to complete sentences with more sophisticated sentence structures or more challenging words, phrases, or constructions. This helps them build on their current language and communication strengths.

> **"Sentence starters are one of the most important things for my students, including our English learners. They give my children a starting point. They have all of the ideas already in their head—they just need that first starting point to take those ideas from their head and put them onto paper or to voice them aloud."**

Promoting Participation in Mathematical Discourse

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Introduction | Sentence Starters and Frames | Partner Work | Full Group Discussion

Partner work promotes student language production—just picture how many more students are talking at once during partner work than during a full-group discussion! Opportunities to express their thinking, try out ideas with a partner, and hear what their partner has to say prior to a class discussion can build students' confidence and give them needed practice in talking about the mathematics before sharing their ideas with the whole class. When students work in pairs productively, they have an opportunity to learn from each other and share new approaches.

> **"When my students worked in pairs, I saw an increase in understanding as well as the linguistic complexity of their answers. It gave each person an opportunity to be an expert and they were able to teach each other. Working in pairs gave them an equal opportunity to be part of the solving, to be part of the thinking, and to be part of the communicating."**

Continued

Promoting Participation in Mathematical Discourse

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Introduction | Sentence Starters and Frames | Partner Work | Full Group Discussion

Tips for productive partner work:

- ☑ To create opportunities for all students to share their ideas, establish a process for partner work that encourages taking turns sharing mathematical thinking, listening carefully when partners share their mathematical thinking, and asking questions to elicit and advance their partners' thinking.
- ☑ Be strategic when pairing students. Consider students' personalities, academic strengths, English language proficiencies, and the goals of the lesson (Chval et al., 2021).
- ☑ Clearly define who each student's partner is so they know whom to work with and can take joint responsibility for working on the task.
- ☑ To fuel productive partner discussions, allow time for students to start to develop their own mathematical ideas individually before moving into pairs or small groups.
- ☑ Use partnerships rather than groups of three or more for discussion when possible because individual students may not have as much of a voice or as much ownership in larger groups.
- ☑ Sometimes, with the best of intentions, peers may assume that the best way to "help" a multilingual student is to dominate the discussion. Position multilingual learners as competent thinkers and not as individuals in need of help. A classroom environment that supports all learners and positions both monolingual and multilingual leaners as competent mathematical thinkers is critical to productive partner work (Chval et al., 2021).
- ☑ Encourage students to use sentence starters and their own diagrams to focus their talk on the mathematical work of the task.
- ☑ Use prompts such as the following to support communication between partners:
  - How could you use your diagram to share your thinking?
  - Have you both had the same amount of time to share your ideas?
  - Listen to your partner. What did you notice that your partner did to solve the problem?
  - Ask your partner: What did you do first? Why was that your first step?
- ☑ Listen for unhelpful patterns in partner work, such as one person dominating the talk, and intervene as needed.

Promoting Participation in Mathematical Discourse

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Introduction | Sentence Starters and Frames | Partner Work | Full Group Discussion

Full-group discussions offer an opportunity for teachers to highlight students' contributions and connect their ideas to one another and to the mathematical goals of the lesson for the benefit of all students. When implemented with equity in mind, a full-group discussion can promote equitable participation and provide an opportunity to hear all students' ideas, including the ideas of multilingual learners.

Tips for facilitating full-group discussions:

- ☑ Encourage all students to share, including multilingual students. When teachers do not call on their multilingual students, those students do not have the same opportunities as others and may feel invisible (Celedon-Pattichis & Ramirez, 2012). Emphasize that students can share beginning ideas and wonderings and can use informal language—they do not need to have the right answer or the right vocabulary word in order to share an idea.
- ☑ Use students' diagrams and solution strategies, including those from multilingual learners, to guide the discussion. Encouraging students to learn from one another and to see the value of one another's ideas supports all students, including multilingual learners.

> "Using diagrams makes it easier to explain my thinking."
>
> -Grade 6 student

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Promoting Participation in Mathematical Discourse

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- Use revoicing as a tool for helping to add clarity and precision to students' ideas. Revoicing is not about correcting students but rather listening carefully so that you can honor their ideas, and then repeating or rephrasing what students say. Revoicing allows you to add to or seek clarity about what students are saying, to highlight particular mathematical ideas that students raise, or to bring academic language and patterns of talk into the discussion in ways that build off students' own ideas. Other key points regarding use of this tool:
  - Don't interrupt a student during their train of thought. Instead, wait for a natural pause in a student's explanation or discussion.
  - Honor and affirm students' ideas whether they are fully correct or not. Restating a student's idea highlights how it is productive for the student to have voiced that idea to move forward the thinking of both the student and the class, regardless of whether the thinking seems correct or complete.
  - Encourage and accept more informal descriptions of concepts rather than wait for a student to come up with exactly the right word. You can then add to the student's thinking by revoicing the idea in ways that bring in more precise language.

> **"I asked one of my students to explain his thinking, which he was willing to do even though he struggled a little bit with the English. We spoke later and he said this was OK and he actually hopes that I'll push him more by calling on him even when he's not raising his hand."**

References

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